

1YGB - MPI - PAGE 11 - QUESTION 1

AS THE CURVE IS GIVEN IN FACTORIZED FORM, WE HAVE THE INTEGRATION LIMITS, BY INSPECTION

$$A_{\text{total}} = \int_{x_1}^{x_2} f(x) dx = \int_{-1}^3 (3-x)(x+1) dx$$

$$= \int_{-1}^3 3x + 3 - x^2 - x dx$$

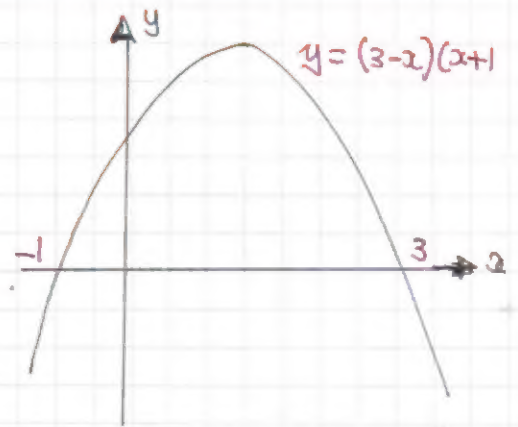
$$= \int_{-1}^3 -x^2 + 2x + 3 dx$$

$$= \left[-\frac{1}{3}x^3 + x^2 + 3x \right]_{-1}^3$$

$$= (-9 + 9 + 9) - \left(+\frac{1}{3} + 1 - 3 \right)$$

$$= 9 - \left(-\frac{5}{3} \right)$$

$$= \frac{32}{3}$$



1YGB - MPI - PAPER M - QUESTION 2

a) If $f(x) = 2^x$ THEN $f(x-3) = 2^{(x-3)}$

HENCE THIS IS A TRANSLATION, IN THE POSITIVE x DIRECTION BY 3 UNITS

b) REWRITING AS FOLLOWS

$$y = 2^{x-3} = 2^x \times 2^{-3} = 2^x \times \frac{1}{2^3} = \frac{1}{8}(2^x)$$

HENCE IF $f(x) = 2^x$ THEN $\frac{1}{8}f(x) = \frac{1}{8}(2^x)$

THIS IS ALSO A VERTICAL STRETCH BY SCALE FACTOR OF $\frac{1}{8}$

-1-

YGB-MPI-PAPER 11 - QUESTION 3

WE DO NOT ACTUALLY NEED THE EXPANSION AS WE ARE ONLY BEING ASKED FOR A SINGLE TERM - THIS WE HAVE

$$(2+3x)^9 = \dots + \binom{9}{5} (2)^4 (3x)^5 + \dots$$

$$\binom{9}{4} (2)^4 (3x)^5 \quad \text{OR} \quad \text{SINCE } \binom{9}{4} = \binom{9}{5}$$

$$= \dots + 126 \times 16 \times 243x^5 + \dots$$

$$= \dots + 489888x^5 + \dots$$

lt 489 888

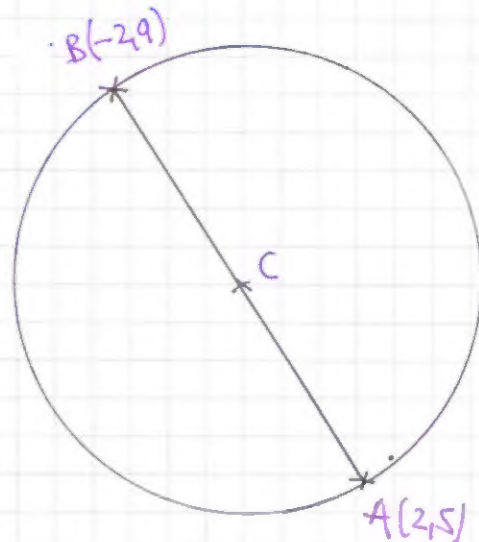
1YGB - MPI - PAPER M - QUESTION 4

- a) THE CENTRE OF THE CIRCLE MUST BE
AT THE MIDPOINT OF AB

$$C = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) = \left(\frac{-2 + 2}{2}, \frac{9 + 5}{2} \right)$$

∴ CENTRE IS AT C(0,7)

THE RADIUS WILL BE THE DISTANCE FROM
A(2,5) TO C(0,7), OR INDEED FROM B TO C



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 2)^2 + (7 - 5)^2} = \sqrt{4 + 4} = \sqrt{8}$$

∴ THE RADIUS IS $\sqrt{8}$

∴ THE REQUIRED EQUATION IS

$$(x - 0)^2 + (y - 7)^2 = (\sqrt{8})^2$$

$$\underline{x^2 + (y - 7)^2 = 8}$$

- b) FIND THE DISTANCE FROM P(1,5) TO THE CENTRE OF THE CIRCLE AT C(0,7)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 1)^2 + (5 - 7)^2} = \sqrt{1 + 4} = \sqrt{5}$$

AS $\sqrt{5} < \sqrt{8}$ THE POINT IS INSIDE

LYGB-MPI-PAPER M-QUESTION 5

$$\cos(2y-35) = 0.891 \quad 0 \leq y < 360$$

$$\arccos(0.891) = 27.000823... \approx 27.0^\circ$$

$$\Rightarrow \begin{cases} 2y-35 = 27.0^\circ \pm 360n \\ 2y-35 = 333.0^\circ \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

\uparrow
 $360-27$

$$\Rightarrow \begin{cases} 2y = 62 \pm 360n \\ 2y = 368 \pm 360n \end{cases}$$

$$\Rightarrow \begin{cases} y = 31 \pm 180n \\ y = 184 \pm 180n \end{cases}$$

LOOKING AT THE REQUIRED RANGE

$$y_1 = 31^\circ$$

$$y_2 = 211^\circ$$

$$y_3 = 184^\circ$$

$$y_4 = 4^\circ$$

1YGB-MPI-PAPER M-QUESTION 6

$$(x^2-x-3)^2-12(x^2-x-3)+27=0$$

A SENSIBLE SUBSTITUTION WILL REDUCE THIS INTO A SIMPLE QUADRATIC

LET $y = x^2 - x - 3$

$$\Rightarrow y^2 - 12y + 27 = 0$$

$$\Rightarrow (y-9)(y-3) = 0$$

$$\Rightarrow y = \begin{matrix} 9 \\ 3 \end{matrix}$$

$$\Rightarrow x^2 - x - 3 = \begin{matrix} 9 \\ 3 \end{matrix}$$

SOLVING EACH QUADRATIC SEPARATELY WE OBTAIN

$$\Rightarrow x^2 - x - 3 = 9$$

$$\Rightarrow x^2 - x - 12 = 0$$

$$\Rightarrow (x-4)(x+3) = 0$$

$$\Rightarrow x = \begin{matrix} 4 \\ -3 \end{matrix}$$

$$\Rightarrow x^2 - x - 3 = 3$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow (x+2)(x-3) = 0$$

$$\Rightarrow x = \begin{matrix} 3 \\ -2 \end{matrix}$$

HENCE THERE ARE 4 REAL SOLUTIONS

$$x = -3, -2, 3, 4$$

YGB - MPI - PAPER M - QUESTION 7

$$f(x) = x^2 - 3x + 7, x \in \mathbb{R}$$

FIRST OBTAIN A SIMPLIFIED EXPRESSION FOR $f(x+h)$

$$\begin{aligned} f(x+h) &= (x+h)^2 - 3(x+h) + 7 \\ &= x^2 + 2xh + h^2 - 3x - 3h + 7 \end{aligned}$$

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{(x^2 + 2xh + h^2 - 3x - 3h + 7) - (x^2 - 3x + 7)}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} \left[\frac{2xh + h^2 - 3h}{h} \right]$$

$$f'(x) = \lim_{h \rightarrow 0} [2x + h - 3]$$

$$f'(x) = 2x - 3$$

AS REQUIRED

IYGB - MPI - PAGE M - QUESTION 8

a) START BY TAKING LOGS (BASE 10) TO BOTH SIDES OF THE EQUATION

$$\Rightarrow y = ab^x$$

$$\Rightarrow \log_{10} y = \log_{10}(ab^x)$$

$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} b^x$$

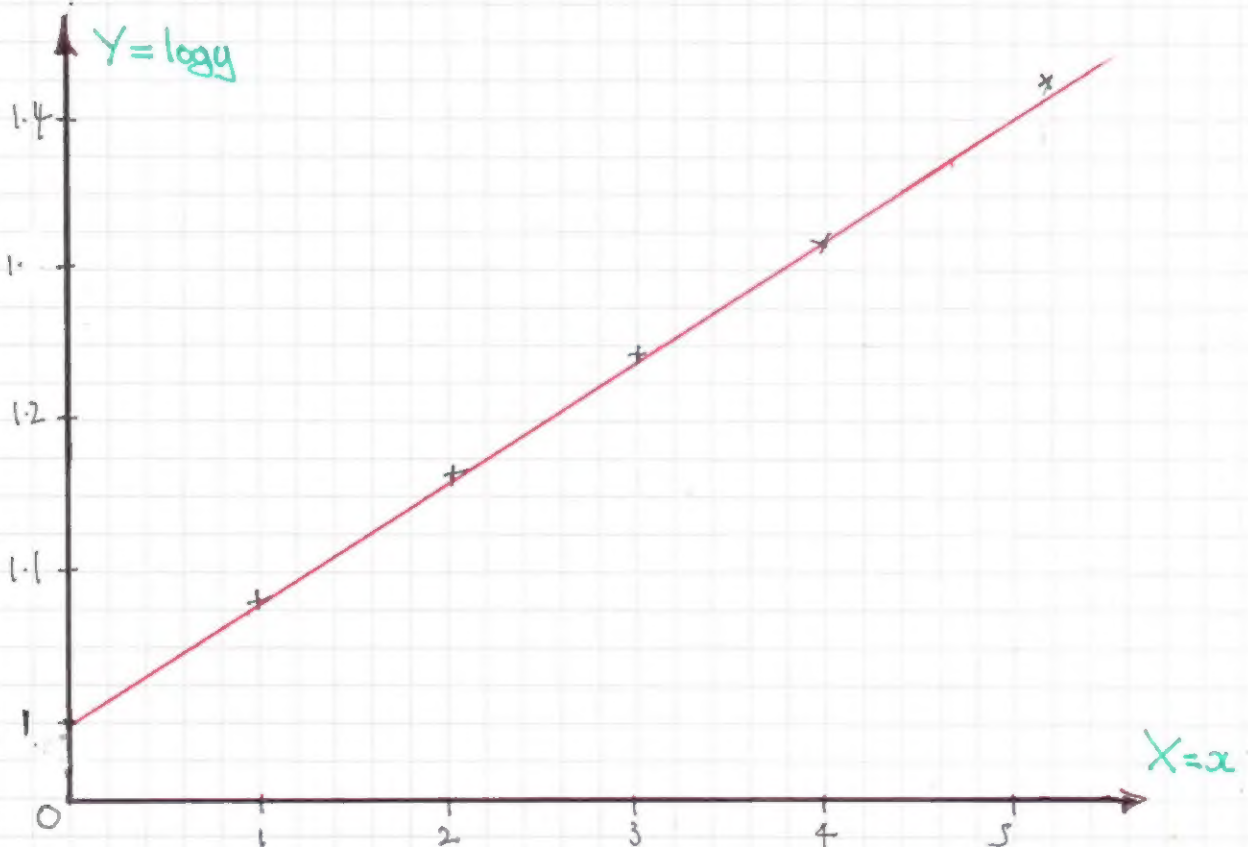
$$\Rightarrow \log_{10} y = \log_{10} a + x \log_{10} b$$

$$\Rightarrow \log_{10} y = (\log_{10} b)x + (\log_{10} a)$$

$$Y = mX + c //$$

b) WE REQUIRE TO PUT $Y = \log_{10} y$ AGAINST $X = x$

$X = x$	1	2	3	4	5
$Y = \log_{10} y$	1.08	1.16	1.24	1.32	1.43



1YGB - MPI - PAPER M - QUESTION 8

AS WE HAVE OBTAINED A STRAIGHT LINE THE ASSUMPTION THAT THE LAW IS THIS FORM IS SUPPORTED

c) THE y INTERCEPT OF THE LINE IS APPROX 0.99

$$c = \log_{10} a$$

$$0.99 = \log_{10} a$$

$$10^{0.99} = a$$

$$a \approx 9.8$$

(2 sf)

THE GRADIENT OF THE LINE IS APPROXIMATELY

$$\frac{1.43 - 0.99}{5 - 0} = 0.088$$

$$m = \log_{10} b$$

$$0.088 = \log_{10} b$$

$$b = 10^{0.088}$$

$$b \approx 1.2$$

2 s.f

d) THE FORMULA NOW READS (APPROXIMATELY)

$$y \approx 9.8 \times 1.2^x$$

$$\text{when } x = 2.5$$

$$y = 9.8 \times 1.2^{2.5}$$

$$y \approx 15$$

(2 sf)

IYGB - MPI - PAPER M - QUESTION 9

SOLVING BY SUBSTITUTION ② INTO ①

$$\left. \begin{array}{l} \textcircled{1} \quad 2x + 2y - z = 2 \\ \textcircled{2} \quad z = x^2 + y^2 \end{array} \right\} \Rightarrow 2x + 2y - (x^2 + y^2) = 2$$

$$\Rightarrow 2x + 2y - x^2 - y^2 = 2$$

$$\Rightarrow 0 = x^2 - 2x + y^2 - 2y + 2$$

$$\Rightarrow 0 = (x-1)^2 - 1 + (y-1)^2 - 1 + 2$$

$$\Rightarrow 0 = (x-1)^2 + (y-1)^2$$

ONLY SOLUTION IS $x=1$ & $y=1$

AND USING $z = x^2 + y^2$, $z=2$

$$\therefore (x, y, z) = (1, 1, 2) //$$

1YGB, MPI, PAPER M - QUESTION 10

$$T = 22 + 50e^{-\frac{1}{10}t}, t > 0$$

T = TEMPERATURE OF DRINK
 t = TIME (IN MINUTES)

a) WHEN $t=0$ $\Rightarrow T = 22 + 50e^0$
 $\Rightarrow T = 22 + 50$
 $\Rightarrow T = 72^\circ\text{C}$

b) WHEN $T = 40$

$$\Rightarrow 40 = 22 + 50e^{-\frac{1}{10}t}$$

$$\Rightarrow 18 = 50e^{-\frac{1}{10}t}$$

$$\Rightarrow \frac{9}{25} = e^{-\frac{1}{10}t}$$

$$\Rightarrow \frac{25}{9} = e^{+\frac{1}{10}t}$$

$$\Rightarrow \frac{1}{10}t = \ln \frac{25}{9}$$

$$\Rightarrow t = 10 \ln \frac{25}{9}$$

$$\Rightarrow t = 8.1732...$$

$$\Rightarrow t \approx 8 \text{ minutes}$$

ii)

DIFFERENTIATE FIRST

$$\Rightarrow T = 22 + 50e^{-\frac{1}{10}t}$$

$$\Rightarrow \frac{dT}{dt} = -\frac{25}{4}e^{-\frac{1}{10}t}$$

WE REQUIRE $\frac{dT}{dt} = -2.5$

$$\Rightarrow -2.5 = -\frac{25}{4}e^{-\frac{1}{10}t}$$

$$\Rightarrow \frac{2}{5} = e^{-\frac{1}{10}t}$$

WE DO NOT ACTUALLY NEED TO SOLVE THIS FULLY AS THIS "LUMP" APPEARS IN THE FORMULA — THIS WE HAVE

$$\Rightarrow T = 22 + 50\left(\frac{2}{5}\right)$$

$$\Rightarrow T = 42^\circ\text{C}$$

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IYGB - MPI - PAPER M - QUESTION 11

$$x^2 + (m+2)x + 4m-7 = 0 \quad x \in \mathbb{R}$$

FOR DISTINCT REAL ROOTS $b^2 - 4ac > 0$

$$a = 1$$

$$b = (m+2)$$

$$c = (4m-7)$$

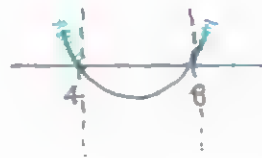
$$\Rightarrow (m+2)^2 - 4 \times 1 \times (4m-7) > 0$$

$$\Rightarrow m^2 + 4m + 4 - 16m + 28 > 0$$

$$\Rightarrow m^2 - 12m + 32 > 0$$

$$\Rightarrow (m-4)(m-8) > 0$$

CRITICAL VALUES $\begin{matrix} & & 4 \\ & \swarrow & \\ & & 8 \end{matrix}$



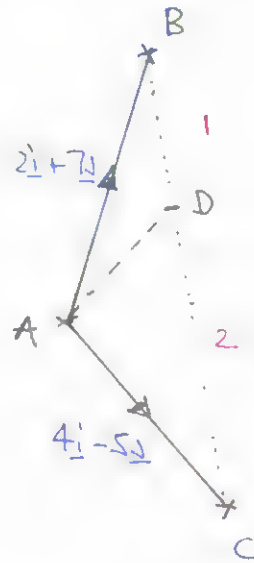
$$m < 4 \text{ OR } m > 8$$

YGB - MPI - PAPER M - QUESTION 12

a) START WITH A DIAGRAM (NOT TO SCALE)

$$\vec{AB} = 2\mathbf{i} + 7\mathbf{j}$$

$$\vec{AC} = 4\mathbf{i} - 5\mathbf{j}$$



IF $|BD| : |DC| = 1 : 2$, THEN

$$\Rightarrow \vec{BD} = \frac{1}{3} \vec{BC}$$

$$\Rightarrow \vec{BD} = \frac{1}{3} (-\vec{BA} + \vec{AC})$$

$$\Rightarrow \vec{BD} = \frac{1}{3} (-2\mathbf{i} - 7\mathbf{j} + 4\mathbf{i} - 5\mathbf{j})$$

$$\Rightarrow \vec{BD} = \frac{2}{3}\mathbf{i} - 4\mathbf{j}$$

b) HENCE WE HAVE, LOOKING AT THE DIAGRAM

$$\Rightarrow \vec{AD} = \vec{AB} + \vec{BD}$$

$$\Rightarrow \vec{AD} = (2\mathbf{i} + 7\mathbf{j}) + (\frac{2}{3}\mathbf{i} - 4\mathbf{j})$$

$$\Rightarrow \vec{AD} = \frac{8}{3}\mathbf{i} + 3\mathbf{j}$$

$$\Rightarrow |\vec{AD}| = \sqrt{\left(\frac{8}{3}\right)^2 + 3^2} = \sqrt{\frac{64}{9} + 9} = \sqrt{\frac{145}{9}} = 4.0138 \dots$$

It APPROX 4

IYGB - MPI - PAPER M - QUESTION 13

a) START WITH THE SKETCH OF EACH OF THE CURVE

$$y = x^2(x-3)$$

$$y = x(x-2)^3$$

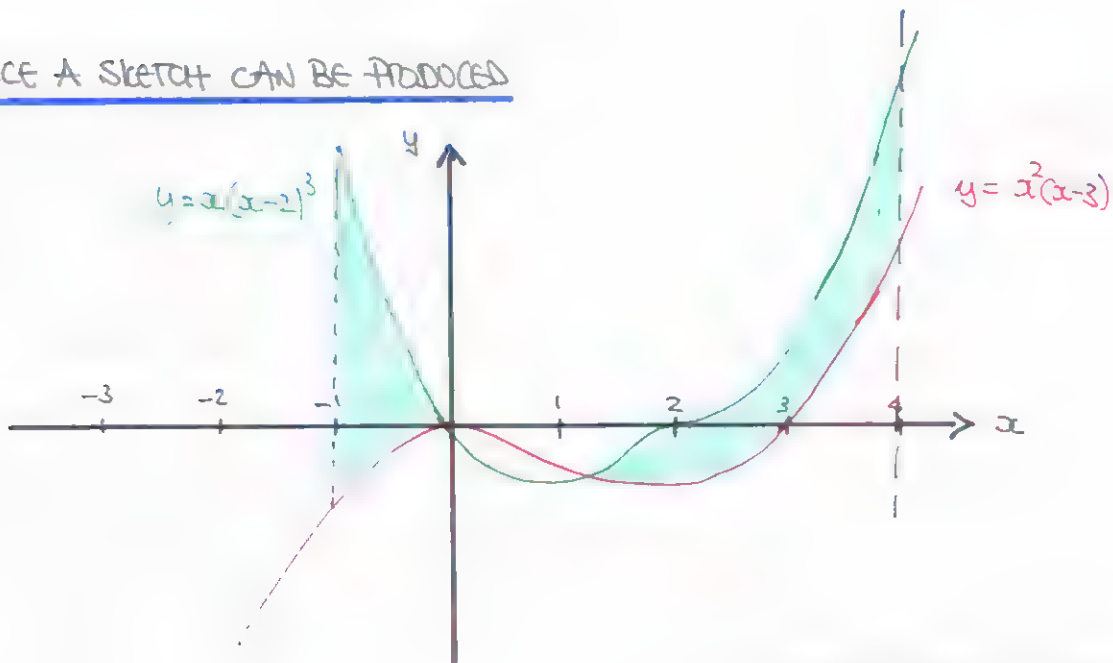
(0,0) Touches

(3,0) Crosses

(0,0) Crosses

(2,0) Stationary Point
of Inflexion

HENCE A SKETCH CAN BE PRODUCED



b)

2 SOLUTIONS AS THERE 2 INTERSECTIONS

$$\begin{cases} x^3 - 3x^2 = x(x-2)^2 \\ x^2(x-3) = x(x-2) \end{cases}$$

c) $y \geq x^3 - 3x^2$ IS THE REGION "ABOVE" THE CURVE IN RED

$y \leq x(x-2)^3$ IS THE REGION "BELOW" THE CURVE IN GREEN

COMBINING WITH $-1 < x < 4$ WE OBTAIN THE REGION ABOVE

(SHADDED IN BLUE)

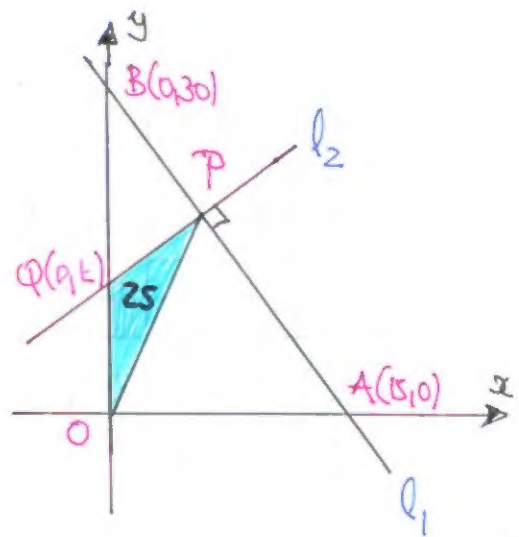
YGB - UPI - PAPER M - QUESTION 14

- a) START BY FINDING THE GRADIENT OF l_1

$$m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 30}{15 - 0} = -2$$

EQUATION OF l_1 USING $(0, 30)$ IS

$$y = 30 - 2x$$



- b) EQUATION OF l_2 , WITH GRADIENT $+\frac{1}{2}$ PASSING THROUGH $Q(0, k)$

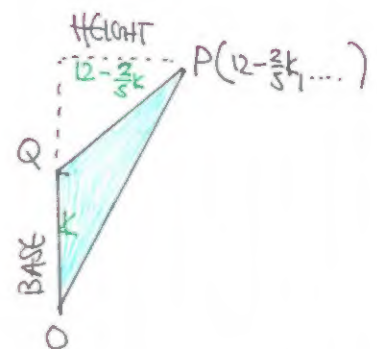
$$y = \frac{1}{2}x + k$$

SOLVING SIMULTANEOUSLY WITH l_1 BY SUBSTITUTION

$$\begin{aligned} \Rightarrow 30 - 2x &= \frac{1}{2}x + k \\ \Rightarrow 60 - 4x &= x + 2k \\ \Rightarrow 60 - 2k &= 5x \\ \Rightarrow x &= 12 - \frac{2}{5}k \end{aligned}$$

- c) AREA OF $\triangle OQP = 25$

$$\begin{aligned} \Rightarrow \frac{1}{2} \times k \times (12 - \frac{2}{5}k) &= 25 \\ \Rightarrow k(12 - \frac{2}{5}k) &= 50 \\ \Rightarrow 12k - \frac{2}{5}k^2 &= 50 \\ \Rightarrow 6k - \frac{1}{5}k^2 &= 25 \\ \Rightarrow 30k - k^2 &= 125 \\ \Rightarrow 0 &= k^2 - 30k + 125 \\ \Rightarrow (k - 5)(k - 25) &= 0 \end{aligned}$$



$$\therefore k = \begin{matrix} 5 \\ 25 \end{matrix}$$

IYGB-MPI-PAPER M - QUESTION 14

NOW IF $k=5$

$$y = \frac{1}{2}x + 5 \quad \& \quad \text{THE } x \text{ CO-ORDINATE OF } P \text{ IS } 12 - \frac{2}{5} \times 5 = 10$$

$$\therefore \text{AREA OF } \triangle OPA = \frac{1}{2} \times 15 \times 10 = 75$$

$$\therefore P(10, 10)$$

$$\therefore \text{AREA OF } OQPA = 75 + 25 = 100$$

AND IF $k=25$

$$y = \frac{1}{2}x + 25 \quad \& \quad \text{THE } x \text{ CO-ORDINATE OF } P \text{ IS } 12 - \frac{2}{5} \times 25 = 2$$

$$\therefore \text{AREA OF } \triangle OPA = \frac{1}{2} \times 15 \times 26 = 195$$

$$\therefore P(2, 26)$$

$$\therefore \text{AREA OF } OQPA = 195 + 25 = 220$$

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IYGB - MPI - PAPER M - QUESTION 15

a) START BY REWRITING THE EQUATION IN INDICIAL FORM, THEN DIFFERENTIATE

$$y = \frac{x^3(5x\sqrt{x} - 128)}{\sqrt{x}} = \frac{5x^4\sqrt{x} - 128x^3}{\sqrt{x}} = \frac{5x^4\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}} - \frac{128x^3}{\sqrt{x}}$$

$$\bullet y = 5x^4 - 128x^{\frac{5}{2}}$$

$$\bullet \frac{dy}{dx} = 20x^3 - 320x^{\frac{3}{2}}$$

$$\bullet \frac{d^2y}{dx^2} = 60x^2 - 480x^{\frac{1}{2}}$$

$$\bullet \frac{d^3y}{dx^3} = \underline{120x - 240x^{-\frac{1}{2}}}$$

b) FOR STATIONARY POINTS $\frac{dy}{dx} = 0$

$$\Rightarrow 20x^3 - 320x^{\frac{3}{2}} = 0$$

$$\Rightarrow x^3 - 16x^{\frac{3}{2}} = 0$$

$$\Rightarrow x^3 = 16x^{\frac{3}{2}}$$

$$\Rightarrow \frac{x^3}{x^{\frac{3}{2}}} = 16$$

$$\Rightarrow \underline{x^{\frac{3}{2}} = 16}$$

$$\Rightarrow (x^{\frac{3}{2}})^{\frac{2}{3}} = 16^{\frac{2}{3}}$$

$$\Rightarrow x^1 = (2^4)^{\frac{2}{3}}$$

$$\Rightarrow \underline{x = 2^{\frac{8}{3}}}$$

SUBSTITUTE INTO Y & TIDY

$$\Rightarrow y = 5x^4 - 128x^{\frac{5}{2}}$$

$$\Rightarrow y = x^{\frac{5}{2}} [5x^{\frac{3}{2}} - 128]$$

$$\Rightarrow y = (2^{\frac{8}{3}})^{\frac{5}{2}} [5 \times \underline{16} - 128]$$

$$\Rightarrow y = 2^{\frac{20}{3}} [80 - 128]$$

$$\Rightarrow y = 2^{6\frac{2}{3}} (-48)$$

$$\Rightarrow y = 2^6 \times 2^{\frac{2}{3}} \times (-48)$$

$$\Rightarrow y = -48 \times 64 \times (2^2)^{\frac{1}{3}}$$

$$\Rightarrow y = \underline{-3072 \times \sqrt[3]{4}}$$

1YGB-MPI-PAPER M - QUESTION 15

c) $\frac{d^2y}{dx^2} = 60x^2 - 480x^{\frac{1}{2}}$

$$\Rightarrow \frac{d^2y}{dx^2} = 60x^{\frac{1}{2}}(x^{\frac{3}{2}} - 8)$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{x^{\frac{3}{2}}=16} = 60 \left(2^{\frac{8}{3}} \right)^{\frac{1}{2}} (16-8) = 60 \times 2^{\frac{4}{3}} \times 8 = 60 \times 2^1 \times 2^{\frac{1}{3}} \times 8$$

$$\Rightarrow \left. \frac{d^2y}{dx^2} \right|_{x^{\frac{3}{2}}=16} = \underline{960 \sqrt[3]{2}}$$

d)

Firstly $\frac{d^2y}{dx^2} = 0$

$$\Rightarrow 60x^2 - 480x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 - 8x^{\frac{1}{2}} = 0$$

$$\Rightarrow x^2 = 8x^{\frac{1}{2}} \quad \left. \begin{array}{l} \\ \end{array} \right\} x \neq 0$$

$$\Rightarrow \frac{x^2}{x^{\frac{1}{2}}} = 8$$

$$\Rightarrow x^{\frac{3}{2}} = 8$$

$$\Rightarrow \left(x^{\frac{3}{2}} \right)^{\frac{2}{3}} = 8^{\frac{2}{3}}$$

$$\Rightarrow x' = (\sqrt[3]{8})^2$$

$$\Rightarrow \underline{x = 4}$$

Finally

$$\Rightarrow \frac{d^3y}{dx^3} = 120x - 240x^{-\frac{1}{2}}$$

$$\Rightarrow \left. \frac{d^3y}{dx^3} \right|_{x=4} = 120 \times 4 - 240 \times 4^{-\frac{1}{2}}$$

$$= 480 - 240 \times \frac{1}{2}$$

$$= 480 - 120 = \underline{360}$$